Research on Calculating Eddy Current Losses in Power Transformer Tank Walls Using Finite Element Method Combined with Analytical Method

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A new analysis method to calculate eddy current losses in large power transformer tank walls is presented. In this model, the perpendicular flux density has been obtained at discrete points on the surface of the upper face of the tank by Finite Element Method (FEM). Then, double Fourier series is adopted to express the obtained flux density by analytical expressions. The coefficients of the analytical expressions are determined by a least mean error with curve fitting technique and optimization algorithm. Based on the Electromagnetic theory and Maxwell equations, the eddy current losses and distribution are obtained by the analytical formulae. The validation of this approach is verified by *TEAM Problem 21 (Model B)*, showing that the calculation results are in good agreement with measured value. Then, it could be extended to calculate the eddy current losses in the tank walls of power transformer.

***Index Terms*—FEM, Double Fourier series, Eddy current losses, Power transformer.**

I. Introduction

WITH the growing capacity of power transformer and increasing voltage rating, the evaluation of stray losses produced by leakage flux is very important for the design of large power transformers. Many researchers have proposed different analysis methods to calculate and reduce the stray losses in flitch plates, frames, and tank walls for eliminating the local overheating and efficiency decreasing [1]-[3]. When it comes to eddy losses calculation of tank walls and other structural parts, due to the limitation of computational resource and nonlinear properties, the results obtained by FEM are not always satisfied in accuracy, especially for the large power transformer's design and optimization.

In this paper, an improved method which combined FEM with Analytical Method is established by considering their respective advantages. The validation of this method is verified through *TEAM Problem 21-B*. Compared with the experimental results, the proposed method is accurate enough for engineering application in eddy losses calculation of the tank walls and other metal structure parts requiring less computational cost.

II. Method Description

A. Establishment of the Analytical Model

In order to describe the proposed analytical model better, we assume one face of the tank wall in a transformer is an individual rectangular solid plate with linear homogeneous property. In Fig. 1, a simplified rectangular steel plate model is given, where *a*, *b*, and *d* is the length, width, and depth, respectively. In the Cartesian coordinate system, magnetic field strength obeys the diffusion equation：

$$\frac{\partial^2 H}{\partial x^2}+\frac{\partial^2 H}{\partial y^2}+\frac{\partial^2 H}{\partial z^2}-j\frac{\omega\mu_0\mu_r}{\rho}H=0 \qquad (1)$$

Where μ_0 is the vacuum permeability, μ_r the relative permeability, *B* the magnetic flux density, and *H* is magnetic field intensity (A/m), ρ the resistivity of tank, respectively. We assume B_z may be expanded as a double Fourier series. Hence, based on [4], the analytical model is expressed as follows:

$$B_z(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}\cos\frac{um\pi x}{a}\cos\frac{vn\pi y}{b} \qquad (2)$$

The coefficients A_{mn} and u , v will be computed according to the following least error fit at all points in the grid and optimization algorithm.

$$\varepsilon^2=\sum_{i=1}^{nx}\sum_{j=1}^{ny}\left[B_z^{'}(x_i,y_j)-B_z(x,y)\right]^2 \qquad (3)$$

On the upper surface of steel, $B'_z(x_i,y_i)$ (*i*=1,2, ⋯*nx*; *j*=1,2,⋯, *ny*) is the normal flux density obtained by FEM at discrete points. Considering the penetration depth, we can evaluate the flux density at any depth in the plate as:

$$B_z(x,y,z)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}\cos\left[\frac{um\pi x}{a}\right]\cos\left[\frac{vn\pi y}{b}\right]\left[\frac{e^{\alpha_{mn}z}-e^{\alpha_{mn}(2d-z)}}{1-e^{2d\alpha_{mn}}}\right] \qquad (4)$$

$$\alpha_{mn}=\sqrt{\left(\frac{um\pi}{a}\right)^2+\left(\frac{vn\pi}{b}\right)^2+j\frac{\omega\mu_0\mu_r}{\rho}}=\beta_{mn}+jY_{mn} \qquad (5)$$

According to $\mathrm{div}B=0$ and $J_z=0$ (boundary condition), we will determine:

$$B_x(x,y,z)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{xmn}\sin\left[\frac{um\pi x}{a}\right]\cos\left[\frac{vn\pi y}{b}\right]\left[\frac{e^{\alpha_{mn}z}+e^{\alpha_{mn}(2d-z)}}{1-e^{2d\alpha_{mn}}}\right] \qquad (6)$$

$$B_y(x,y,z)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{ymn}\cos\left[\frac{um\pi x}{a}\right]\sin\left[\frac{vn\pi y}{b}\right]\left[\frac{e^{\alpha_{mn}z}+e^{\alpha_{mn}(2d-z)}}{1-e^{2d\alpha_{mn}}}\right] \qquad (7)$$

Where A_{xmn} and A_{ymn} are calculated by (8)-(9):

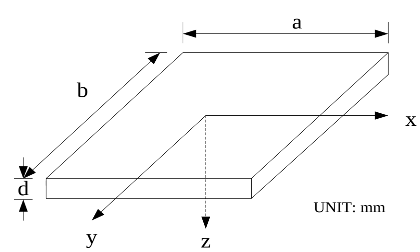


Fig.1. Simplified rectangular steel plate.

$$A_{xmn} = -\frac{\alpha_{mn}A_{mn}}{\pi}\frac{umab^2}{(vna)^2+(umb)^2} \tag{8}$$

$$A_{ymn} = -\frac{\alpha_{mn}A_{mn}}{\pi}\frac{vna^2b}{(vna)^2+(umb)^2} \tag{9}$$

By using the Electromagnetic theory and Maxwell equations, the induced eddy current could be determined when the magnetic field intensity cross the steel plate.

From $\nabla \times H = J$, we will get

$$\left.\begin{aligned} J_x &= \frac{1}{\mu}\left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right) \\ J_y &= \frac{1}{\mu}\left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right) \\ J_z &= 0 \quad \text{(boundary condition)} \end{aligned}\right\} \tag{10}$$

After the flux density has been determined from (3)-(9), $J(x,y,z)$ is easily got from (10).

$$J_x = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} J_{xmn}\cos\left[\frac{um\pi x}{a}\right]\sin\left[\frac{vn\pi y}{b}\right]\left[\frac{e^{\alpha_{mn}z} - e^{\alpha_{mn}(2d-z)}}{1-e^{2d\alpha_{mn}}}\right] \tag{11}$$

$$J_y = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} J_{ymn}\sin\left[\frac{um\pi x}{a}\right]\cos\left[\frac{vn\pi y}{b}\right]\left[\frac{e^{\alpha_{mn}z} - e^{\alpha_{mn}(2d-z)}}{1-e^{2d\alpha_{mn}}}\right] \tag{12}$$

Where J_{xmn} and J_{ymn} will be explained in extended paper.

Considering the induced eddy current has an impact on magnetic field, we need to constantly modify the size of surface magnetic density. Iterative criterion depends on that the difference of the first and second eddy current density is less than 10^{-3}. Therefore, coefficients *m*, *n* will be determined in infinite series. These formulae, with simple form and good accuracy, are easy to be applied in engineering practice.

B. Verified with TEAM Problem 21 (Model B)

Firstly, the value of the radial magnetic flux density on the steel plate surface can be obtained by FEM. In Fig. 2, it is obviously that the flux density simulation result and experiment result [5] in specified locations are consistent. Furthermore, the eddy losses will be found readily in the whole steel plate as (13). Owing to the hysteresis loss W_h is treated to be as a function of the peak value of the flux density B_m, the hysteresis loss will be calculated by FEM with (14).

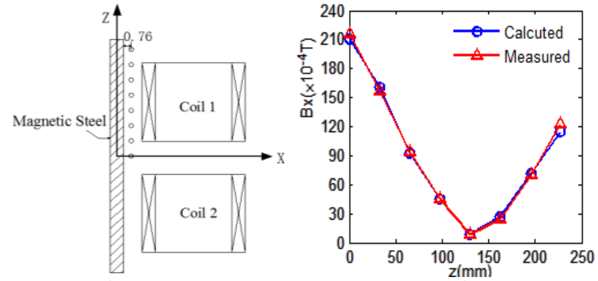
$$P = \frac{1}{2\sigma}\int_0^d \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} \left(\left|J_x^2\right| + \left|J_y^2\right|\right) dxdydz \tag{13}$$

$$W_h = \sum_{i=1}^{N(i)} W_h^{(i)}\left(B_m^{(i)}\right)\rho V(i) \tag{14}$$

TABLE I

COMPARISON OF COMPUTATION RESULT OF STRAY LOSS IN P21-B UNDER DIFFERENT METHOD (UNIT: W)

Calculated method	Total loss	Eddy loss	Hysteresis loss	CPU time/s
FEM	12.04	8.10	3.94	20856
Proposed method	12.56	8.62	3.94	15972
Analytical method[4]	12.33	12.33	-	-
Measured results	11.97	-	-	-



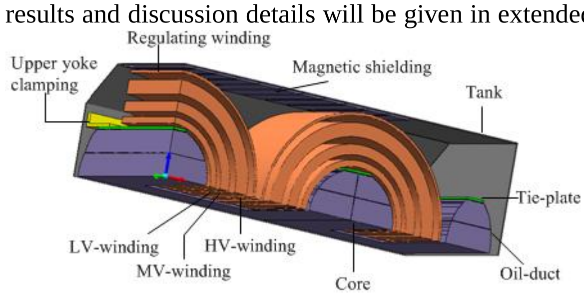
(a) Specific locations (b) Compared the value of measured B_x with the calculated

Fig.2. Team Problem 21-B model.

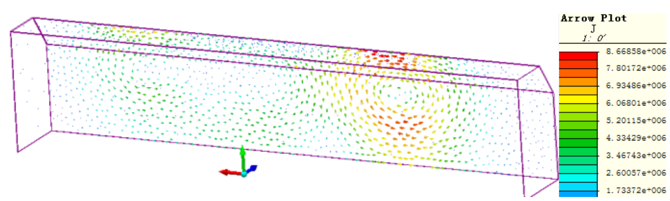
Where σ is the conductivity of tank, J the effective value of the current density, and $W_h^{(i)}$ denotes the dc hysteresis loss the incident flux density (W/kg), $B_m^{(i)}$ is the peak value of the flux density, ρ the density of the steel plate, $V(i)$ the volume of an element, and $N(i)$ is the total number of elements, respectively. Table I lists the compared results.

III. Results

The three-phase five column power transformer simulation model of OSFPS9-360MVA/330GY is shown in Fig. 3. More results and discussion details will be given in extended paper.



(a) One-eighth of the simulation model.



(b) Predicted distribution of eddy current density in a quarter of tank walls

Fig.3. The simulation model of three-phase five column power transformer.

IV. References

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